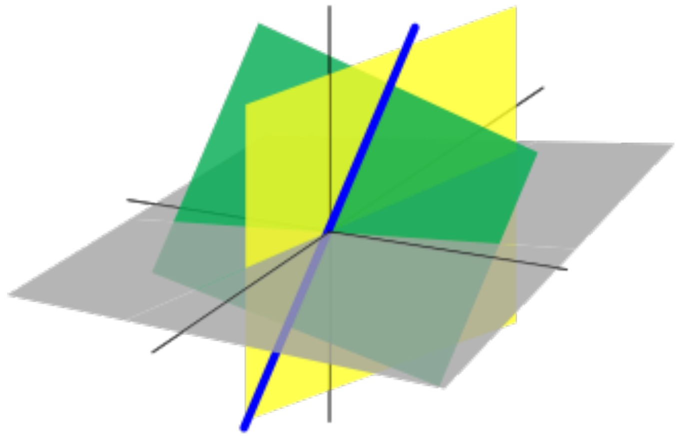

LINVIEW: Incremental View Maintenance for Complex Analytical Queries

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ÉCOLE POLYTECHNIQUE
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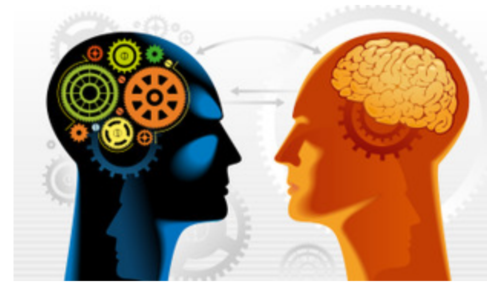
Big Data Analytics

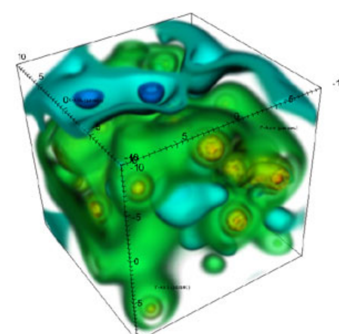
Simple (SQL) Analytics

Data Warehouses (OLAP)

Complex (non-SQL) Analytics



Machine Learning



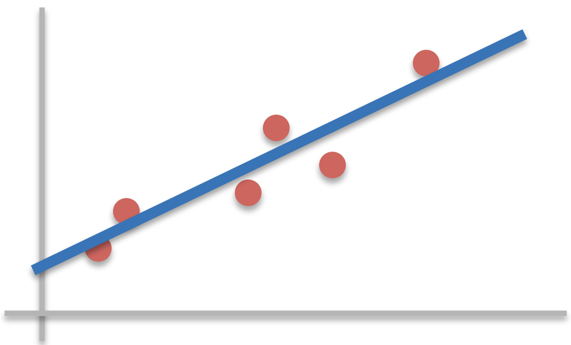
Scientific Computing

Data Mining

Complex Analytical Queries

- Often expressed as linear algebra on array data

    Example: Ordinary Least Squares



$$Y = X\beta$$

$$\beta^* = (X^T X)^{-1} X^T Y$$

Re-evaluating complex queries on every (small) change is inefficient
=> Do it incrementally!

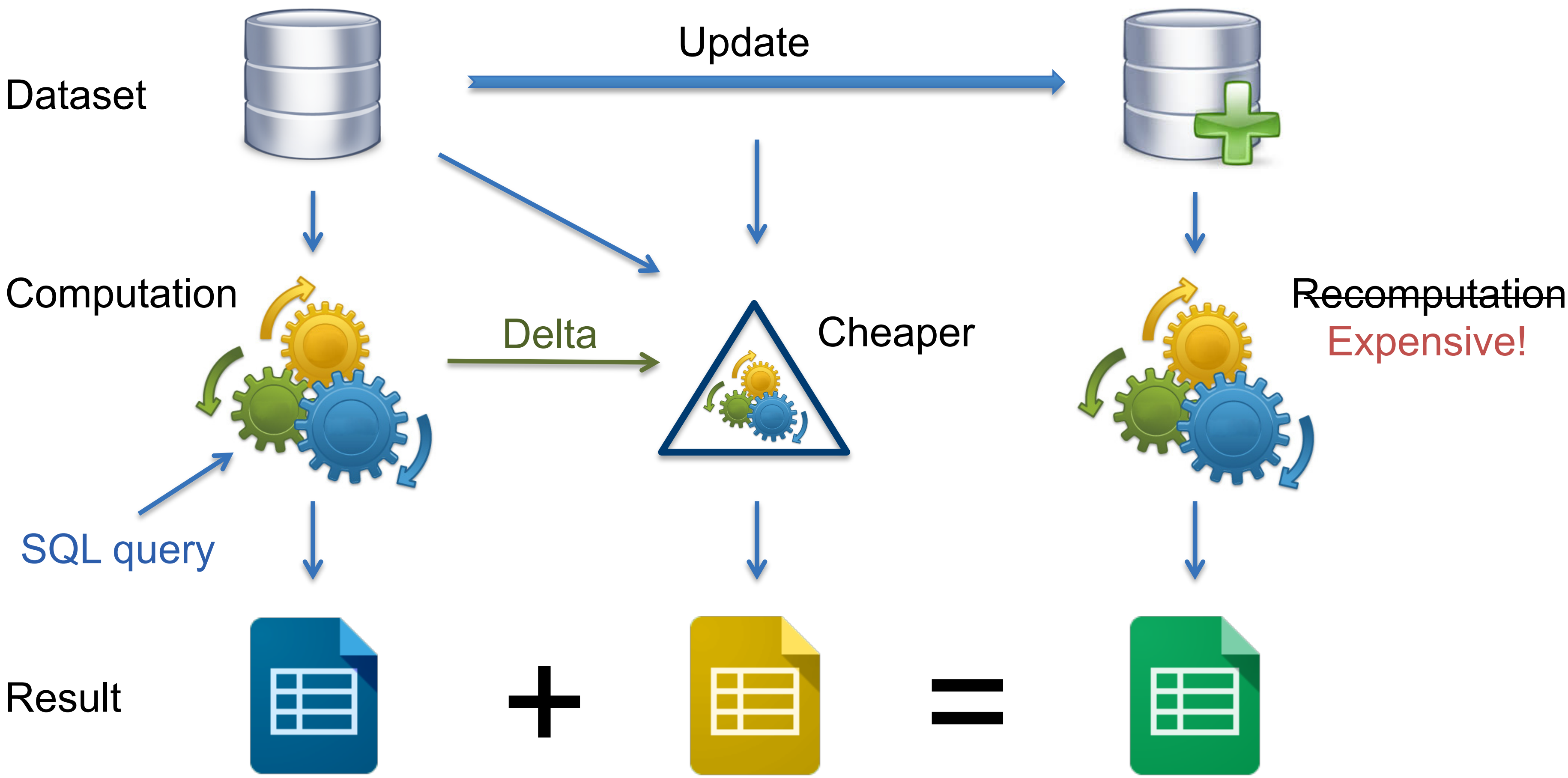
- Multidimensional arrays

    HIGH DIMENSIONAL: Data processing is increasingly expensive

    DYNAMIC: Continuously changing, evolve through small changes (e.g., user's Internet activity)

- Users want frequently fresh views of data

Incremental Processing



Incremental View Maintenance (IVM) in DBMS (Oracle, DB2, PostgreSQL, …)

LINVIEW

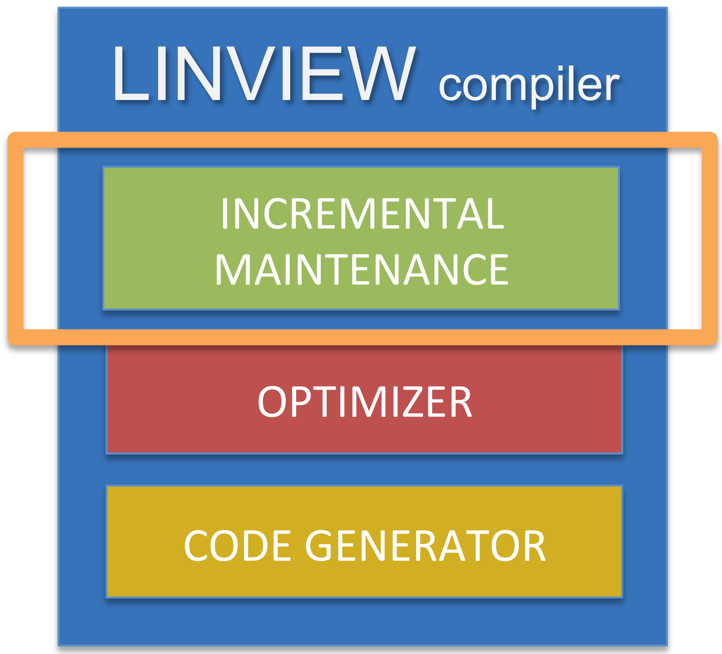
Incremental evaluation of (iterative) linear algebra programs

APL-style programs

For instance:
MATLAB, R, Octave

Matrix operations
$(+/-,\ *,\ A^T,\ A^{-1})$

Basis of ML algos



Incremental programs

Exec over dynamic data

Different runtimes
(Spark, Octave)

Example: Matrix Powers A^4

$$\underset{A}{\begin{bmatrix}0&-1&5\\3&4&1\\-2&1&-3\end{bmatrix}} + \underset{\Delta A}{\begin{bmatrix}0&0&0\\0&0&-2\\0&0&0\end{bmatrix}} = \underset{A + \Delta A}{\begin{bmatrix}0&-1&5\\3&4&-1\\-2&1&-3\end{bmatrix}}$$

Delta Matrix

$B = A\,A$
$C = B\,B$
$O(n^3)$

Incremental Program
$O(n^2)$

$\Delta B = \ ...$
$\Delta C = \ ...$
$B \mathrel{+}= \Delta B$
$C \mathrel{+}= \Delta C$

$A = A + \Delta A$
$B = A\,A$
$C = B\,B$
$O(n^3)$

$$\underset{C}{\begin{bmatrix}35&-47&224\\154&254&64\\-87&51&-96\end{bmatrix}} + \underset{\Delta C}{\begin{bmatrix}22&4&0\\-42&-54&-120\\-6&-12&16\end{bmatrix}} = \underset{C + \Delta C}{\begin{bmatrix}57&-43&224\\112&200&-56\\-93&39&-80\end{bmatrix}}$$

IVM of Linear Algebra

Original Program (Expensive)

```
ON UPDATE A BY ΔA:
    A += ΔA
    B = A A
    C = B B
```

$O(n^3)$

Incremental Program (Cheap)

```
ON UPDATE A BY ΔA:
    ΔB = A(ΔA) + (ΔA)A + (ΔA)(ΔA)
    ΔC = B(ΔB) + (ΔB)B + (ΔB)(ΔB)
    A += ΔA
    B += ΔB
    C += ΔC
```

$O(n^2)$

… when ΔA is "simple"

How to

… derive delta expressions?

… evaluate delta expressions?

… represent delta expressions?

Delta Derivation

- Exploits properties of matrix operations
  (e.g., distributivity of matrix multiplication over addition)

  Example:

  $B[A] = A\,A$ (consider B as a function of A)

  $$\begin{aligned}\Delta B[A, \Delta A] &= B[A + \Delta A] - B[A] \\ &= (A + \Delta A)(A + \Delta A) - A\,A \\ &= A(\Delta A) + (\Delta A)A + (\Delta A)(\Delta A)\end{aligned}$$

- The Sherman–Morrison formula for maintaining $(A + \Delta A)^{-1}$

Delta Evaluation: The Avalanche Effect

$$\underbrace{\begin{bmatrix} 0 & -1 & 5 \\ 3 & 4 & 1 \\ -2 & 1 & -3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}}_{\Delta A} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}}_{\Delta A} \underbrace{\begin{bmatrix} 0 & -1 & 5 \\ 3 & 4 & -1 \\ -2 & 1 & -3 \end{bmatrix}}_{A \,+\, \Delta A} = \underbrace{\begin{bmatrix} 0 & 0 & 2 \\ 4 & -2 & -2 \\ 0 & 0 & -2 \end{bmatrix}}_{\Delta B}$$

$$\underbrace{\begin{bmatrix} -13 & 1 & -16 \\ 10 & 14 & 16 \\ 9 & 3 & 0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} 0 & 0 & 2 \\ 4 & -2 & -2 \\ 0 & 0 & -2 \end{bmatrix}}_{\Delta B} + \underbrace{\begin{bmatrix} 0 & 0 & 2 \\ 4 & -2 & -2 \\ 0 & 0 & -2 \end{bmatrix}}_{\Delta B} \underbrace{\begin{bmatrix} -13 & 1 & -14 \\ 14 & 12 & 14 \\ 9 & 3 & -2 \end{bmatrix}}_{B \,+\, \Delta B} = \underbrace{\begin{bmatrix} 22 & 4 & 0 \\ -42 & -54 & -120 \\ -6 & -12 & 16 \end{bmatrix}}_{\Delta C}$$

A single-entry change contaminates the whole output => $\Omega(n^2)$

↳ Delta computation involves $O(n^3)$ matrix multiplication

↳ IVM loses its performance benefit over re-evaluation

How to confine the avalanche effect?

Delta Representation

- Deltas as single matrices
  - ✗ quickly escalate to full matrices, involve *$O(n^3)$* ops

- Insight: delta matrices have low ranks
  - ✓ represent as vector outer products

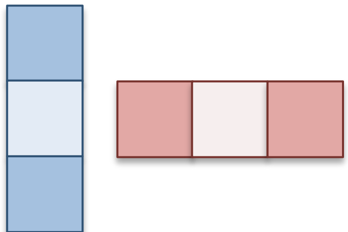
$$\Delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -2 \end{bmatrix} = u\ v^T$$

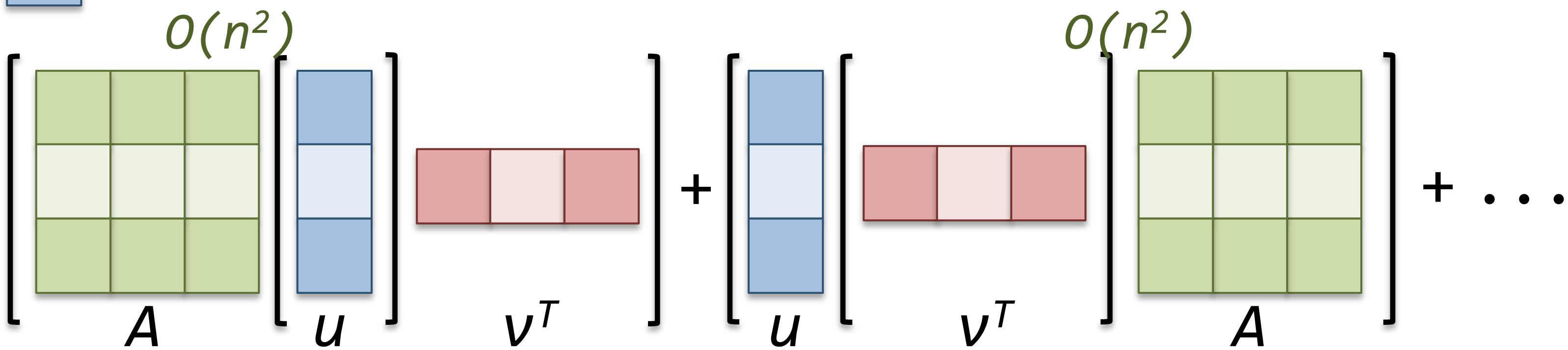
- Factored representation admits efficient evaluation

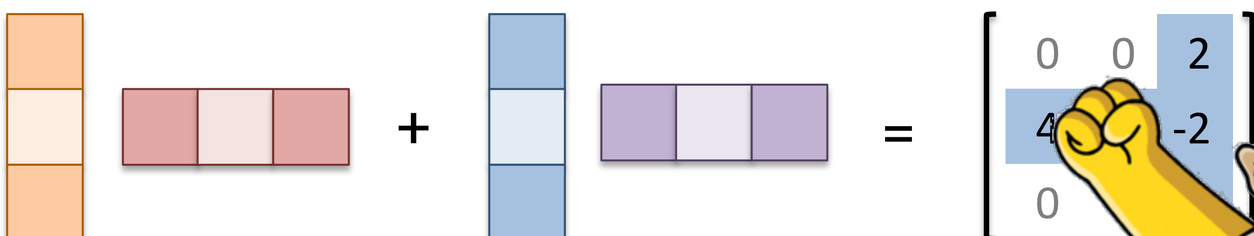
Revisited: Matrix Powers A^4

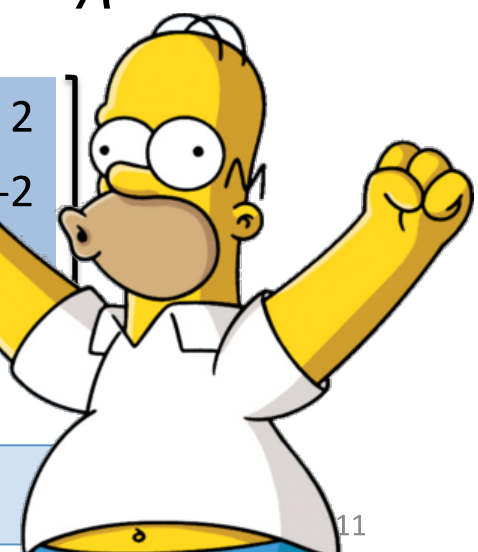
rank-s, efficient when s << n

ΔA is a ~~rank-1~~ update
(e.g., changes of one row/column)

$\Delta A =$  $= u\, v^T$

$\Delta B =$ 

$=$ 

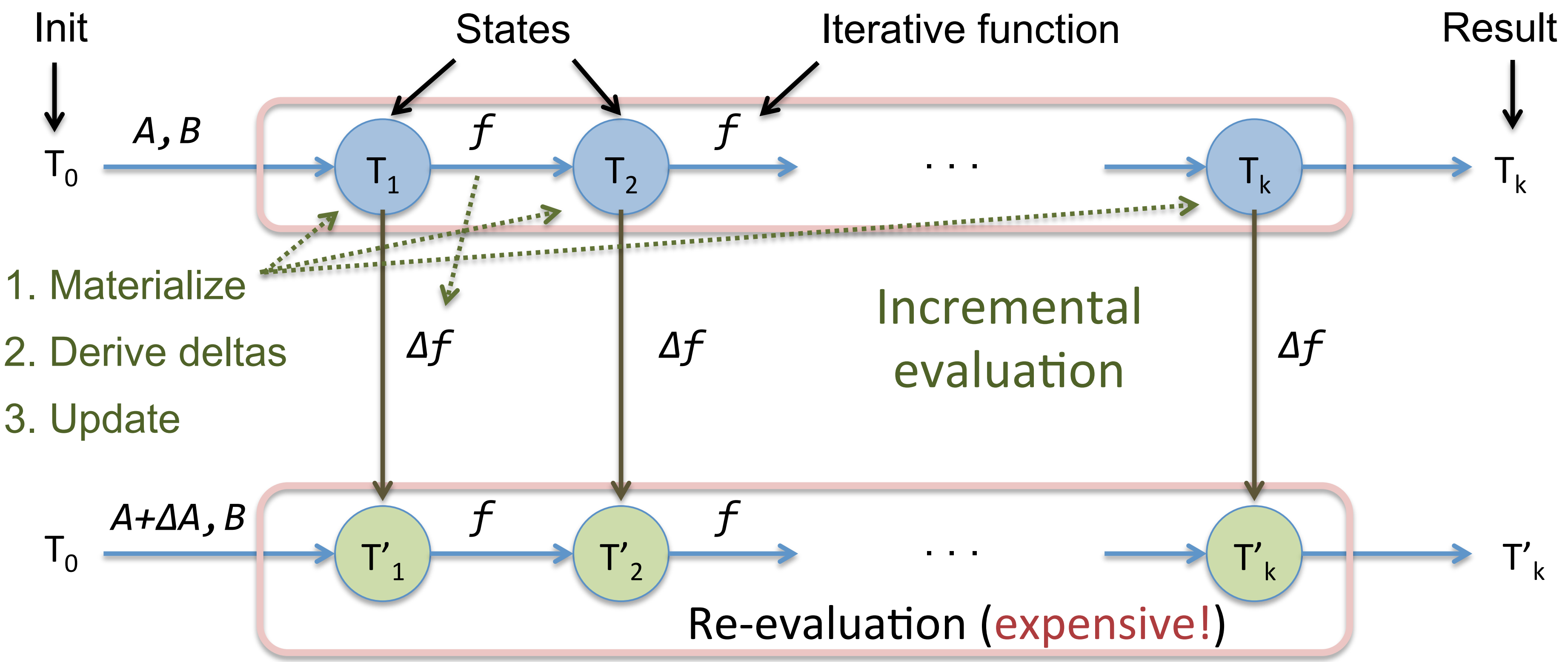


ΔC = a sum of 4 outer products

Delta computation involves only $O(n^2)$ operations!

IVM of Iterative Programs

- Many programs in practice converge within only a few iterations (e.g., 80.7% of pages in PageRank converge in less than 15 iterations[1])
- Example: $T_i = f(A, B, T_{i-1}) = A\,T_{i-1} + B$



[1] S. Kamvar, T. Haveliwala, and G. Golub. Adaptive methods for the computation of PageRank. Technical report, Stanford, 2003

Time Complexity

(rank-1 updates, big-O notation)

	Re-evaluation	Incremental maintenance
Ordinary Least Squares	n^3	n^2
Matrix Powers A^k	$n^3 \log k$	$n^2 k$
$T_{i+1} = A T_i + B$ where $T = (n \times n)$	$n^3 \log k$	$n^2 k$
$T_{i+1} = A T_i + B$ where $T = (n \times 1)$	$n^2 k$	$n^2 k$

A – dimension (n x n)

k – number of iterations

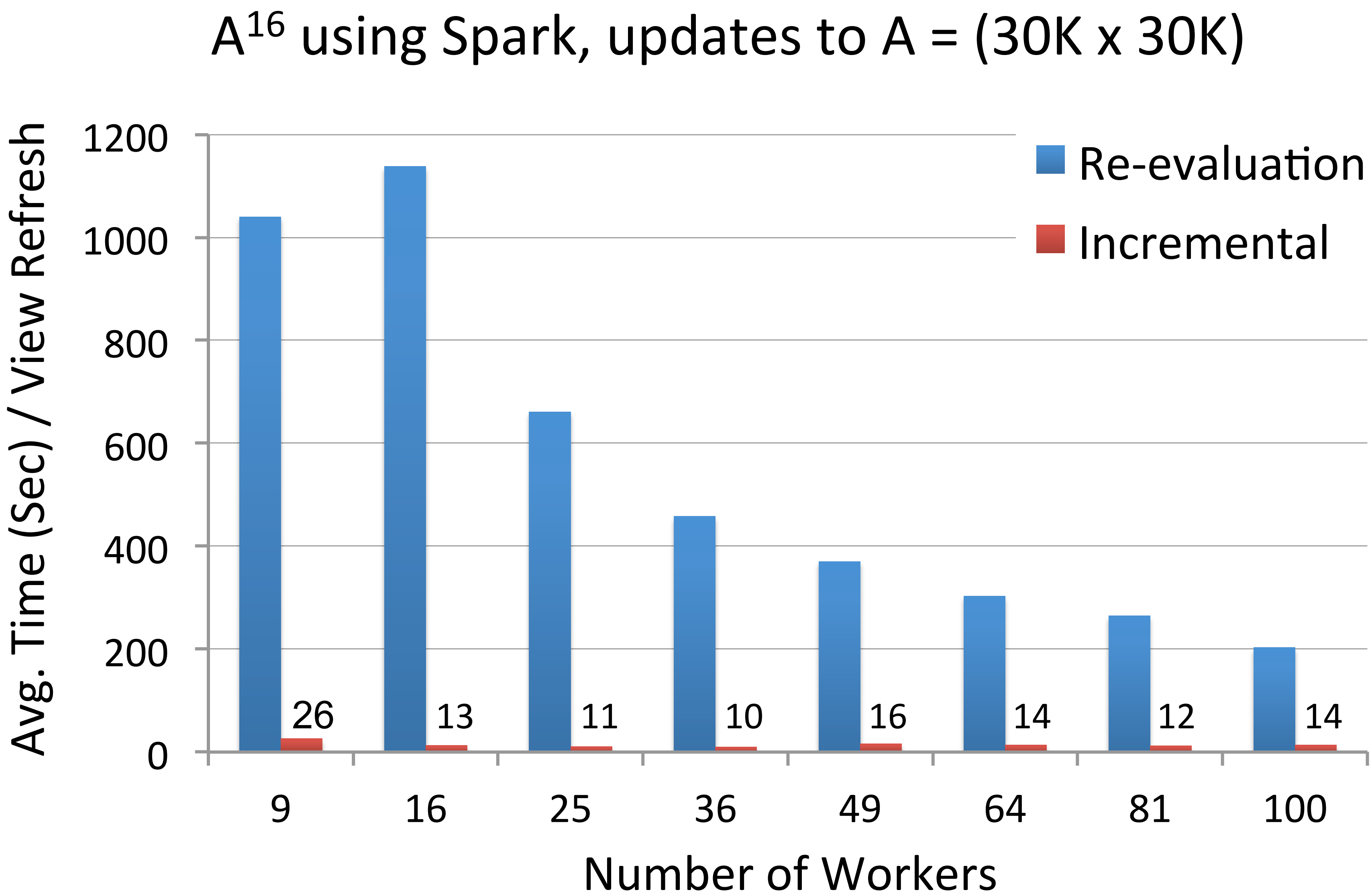
IVM has lower time complexity in most cases!

But increases memory consumption (log *k* times, details in paper)

Experimental Setup

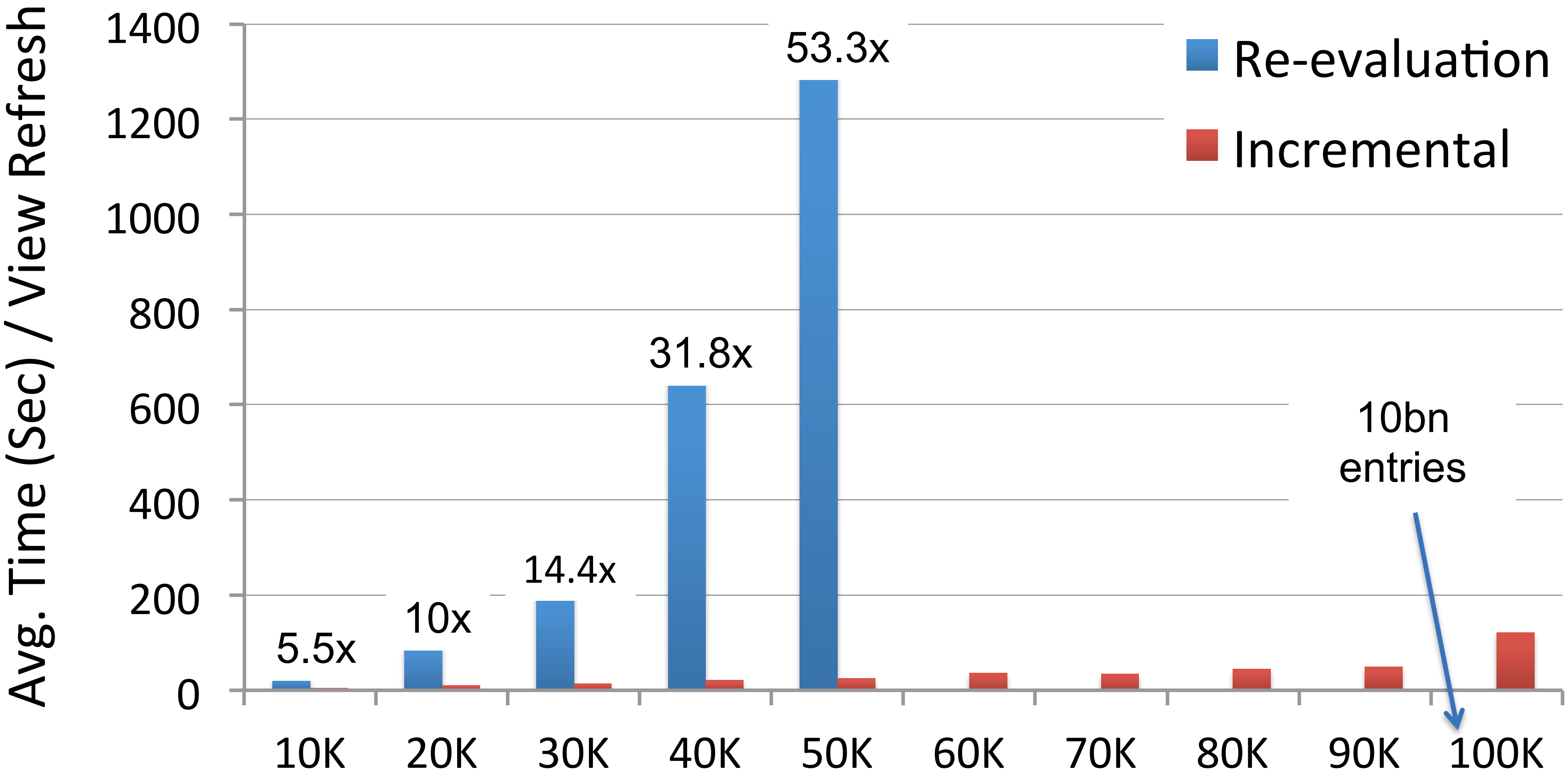
- Analytics: OLS, matrix powers, GD for lin. regression, …
- Apache Spark
  - EC2 cluster: 100 workers (8 vCPUs, 13.6GB RAM, 10GbE)
- GNU Octave
  - 2 x 2.66GHz 6-Core Intel Xeon, 64GB RAM
- Randomly generated dense matrices
  - Preconditioned for numerical stability
- Stream of rank-1 updates
  - Each update affects one row of the input matrix

Matrix Powers – Scalability (nodes)



Matrix Powers – Scalability (dimension)

A^{16} using 100 Spark workers, updates to A = (n x n)

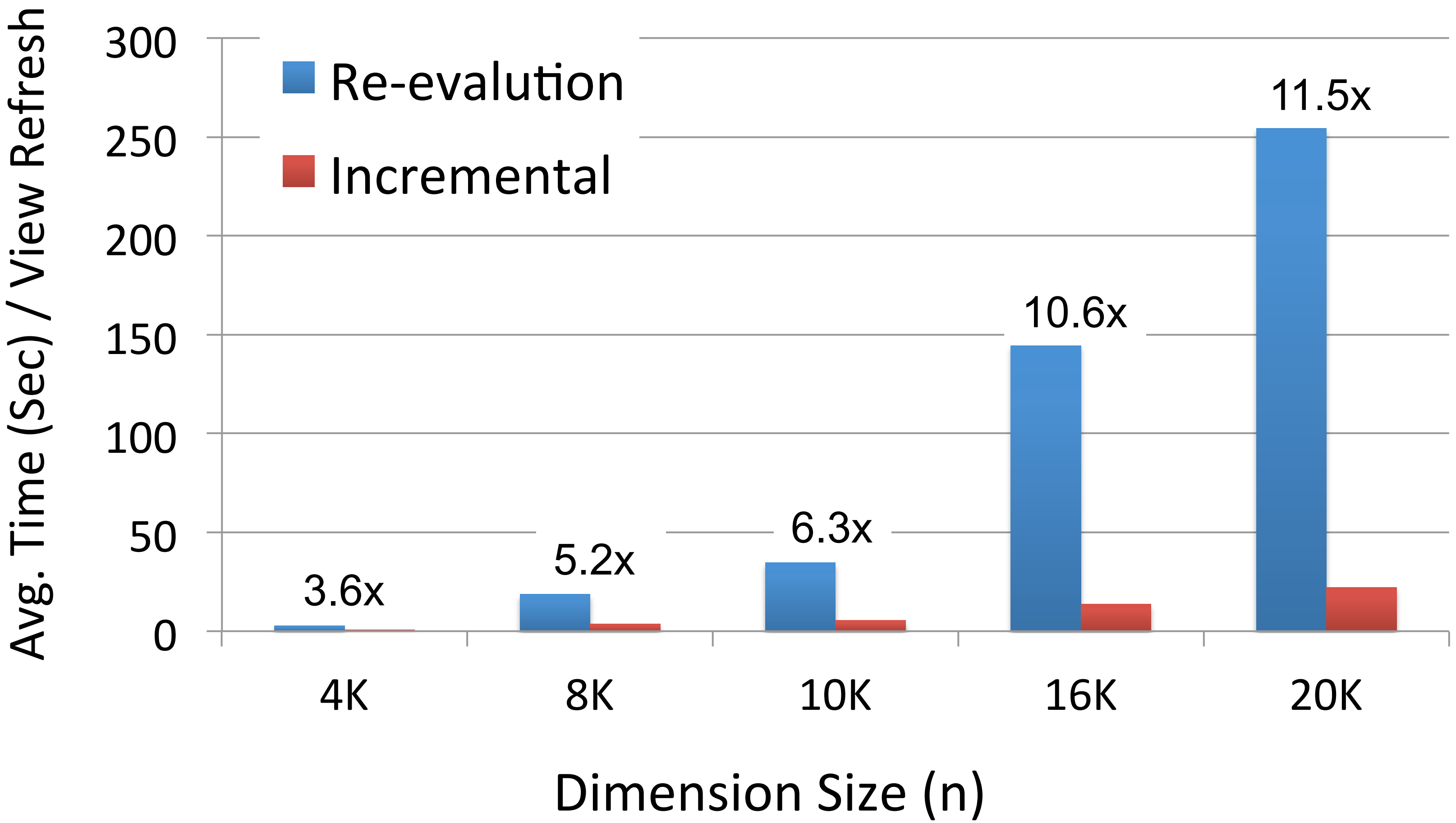


The performance gap increases with higher dimensionality!

Ordinary Least Squares

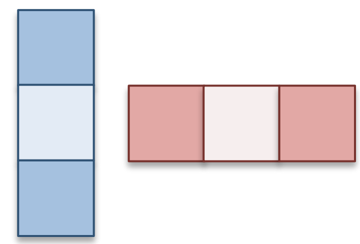
$$\beta^* = (X^T X)^{-1} X^T Y$$

GNU Octave, updates to $X = (n \times n)$, $\beta^*, Y = (n \times 1)$



LINVIEW: Recap

- Incremental computation of analytical queries expressed as linear algebra programs
- Factored delta representation
  - As (sums of) vector outer products
  - Confines the avalanche effect
  - Admits efficient evaluation

$\Delta A =$ 

- IVM has lower time complexity than re-evaluation
  - Can outperform re-evaluation by orders of magnitude

http://data.epfl.ch/linview